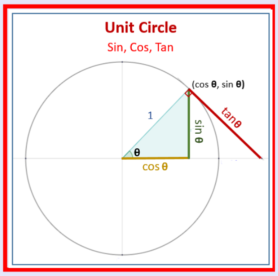
Math 241

Winter 2024

Lecture 11



Feb 19-8:47 AM

$$\sin(A-B) = \sin A\cos B - \cos A\sin B$$

$$\cos(A-B) = \cos A\cos B + \sin A\sin B$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\cdot\tan B}$$

verify $\cos\left(\frac{\pi}{2}-\alpha\right) = \sin\alpha$

$$\cos\left(\frac{\pi}{2}-\alpha\right) = \underbrace{\cos\frac{\pi}{2}}_{0}\cos\alpha + \underbrace{\sin\frac{\pi}{2}}_{1}\sin\alpha$$

$$= \underbrace{0\cdot\cos\alpha}_{0} + \underbrace{1\cdot\sin\alpha}_{\sin\alpha}$$

$$= 0 + \sin\alpha = \boxed{\sin\alpha}$$

Verify $\sin(90^\circ-\alpha) = \cos\alpha$

$$\sin(90^\circ-\alpha) = \underbrace{\sin 90^\circ}_{1}\cos\alpha - \underbrace{\cos 90^\circ}_{0}\sin\alpha$$

$$= 1\cdot\cos\alpha - 0\cdot\sin\alpha = \boxed{\cos\alpha}$$

Show $\tan(90^\circ-\alpha) = \cot\alpha$

$$\tan(90^\circ-\alpha) = \frac{\tan 90^\circ - \tan\alpha}{1 + \tan 90^\circ\cdot\tan\alpha}$$

but $\tan 90^\circ$ is undefined.

$$\tan(90^\circ-\alpha) = \frac{\sin(90^\circ-\alpha)}{\cos(90^\circ-\alpha)} = \frac{\cos\alpha}{\sin\alpha} = \boxed{\cot\alpha}$$

Jan 22-8:03 AM

Complementary angles & Cofunctions:

$\sin(90^\circ - x) = \cos x$ | $\csc(90^\circ - x) = \sec x$

$\cos(90^\circ - x) = \sin x$ | $\sec(90^\circ - x) = \csc x$

$\tan(90^\circ - x) = \cot x$ | $\cot(90^\circ - x) = \tan x$

Find x:

$\sin(3x - 10) = \cos(x + 40)$

Cofunctions $\rightarrow$ $3x - 10 + x + 40 = 90$

$4x + 30 = 90$

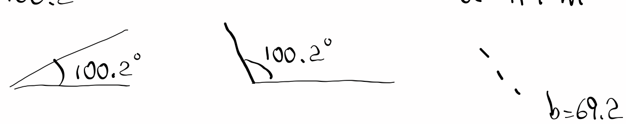
$4x = 60$

$\boxed{x = 15}$

$3x - 10 = 3(15) - 10 = 45 - 10 = \boxed{35^\circ}$ | $x + 40 = 15 + 40 = \boxed{55^\circ}$

$\alpha = 100.2^\circ$

$a = 117$ m



Jan 22-8:13 AM

Find x:

$\tan(45 - 3x)^\circ = \cot(15 - 2x)^\circ$

when cofunctions are equal, angles must be complementary.

$45 - 3x + 15 - 2x = 90$

$60 - 5x = 90$

$-5x = 30$ $\qquad x = -6$

$45 - 3x = 45 - 3(-6) = 45 + 18 = \boxed{63^\circ}$

$15 - 2x = 15 - 2(-6) = 15 + 12 = \boxed{27^\circ}$

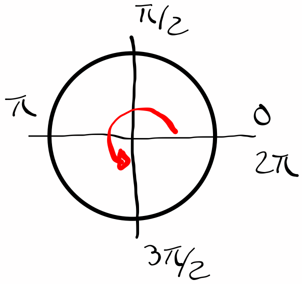
$\tan 63^\circ = 1.963$ ✓

$\cot 27^\circ = \dfrac{1}{\tan 27^\circ} = 1.963$ ✓

Jan 22-8:23 AM

Verify $\sec\left(\frac{3\pi}{2}-x\right) = -\csc x$ ✓

$$\sec\left(\frac{3\pi}{2}-x\right) = \frac{1}{\cos\left(\frac{3\pi}{2}-x\right)}$$



$$= \frac{1}{\cancelto{0}{\cos\frac{3\pi}{2}}\cos x + \cancelto{-1}{\sin\frac{3\pi}{2}}\sin x}$$

$$= \frac{1}{-\sin x} = -\frac{1}{\sin x} = \boxed{-\csc x}$$

Jan 22-8:30 AM

Find the exact value of $\tan\frac{11\pi}{12}$.

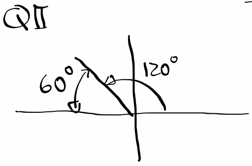
$$\left(\frac{11\cancel{\pi}}{\cancel{12}_{1}}\cdot\frac{\cancel{180}^{15}}{\cancel{\pi}}\right)^\circ = 165^\circ = 120^\circ + 45^\circ$$

$-2+\sqrt{3}$

$$\tan\frac{11\pi}{12} = \tan(120^\circ + 45^\circ)$$

$$= \frac{\tan 120^\circ + \cancelto{1}{\tan 45^\circ}}{1 - \tan 120^\circ \cdot \cancelto{1}{\tan 45^\circ}} =$$

QII



$$= \frac{-\tan 60^\circ + 1}{1 - (-\tan 60^\circ)\cdot 1} = \frac{-\sqrt{3}+1}{1+\sqrt{3}}$$

$$= \frac{1-\sqrt{3}}{1+\sqrt{3}}\cdot\frac{1-\sqrt{3}}{1-\sqrt{3}}$$

$$= \frac{1-2\sqrt{3}+3}{1-3} = \frac{4-2\sqrt{3}}{-2} =$$

$$= \frac{\cancel{2}(-2+\sqrt{3})}{\cancel{-2}} = \boxed{-2+\sqrt{3}}$$

Jan 22-8:35 AM

Simplify $\tan(180^\circ + \theta)$

$$= \frac{\sin(180^\circ + \theta)}{\cos(180^\circ + \theta)}$$

$$= \frac{\sin 180^\circ \cos\theta + \cos 180^\circ \sin\theta}{\cos 180^\circ \cos\theta - \sin 180^\circ \sin\theta}$$

$\sin 180^\circ \to 0$, $\cos 180^\circ \to -1$

$$= \frac{-\sin\theta}{-\cos\theta} = \boxed{\tan\theta}$$

without calc, find

$$\frac{\tan 100^\circ + \tan 80^\circ}{1 - \tan 100^\circ \cdot \tan 80^\circ} = \tan(100^\circ + 80^\circ)$$

$$= \tan 180^\circ = \frac{\sin 180^\circ}{\cos 180^\circ}$$

$$\frac{\tan A + \tan B}{1 - \tan A \cdot \tan B} = \tan(A+B)$$

$$= \frac{0}{-1} = \boxed{0}$$

Jan 22-8:45 AM

More identities:

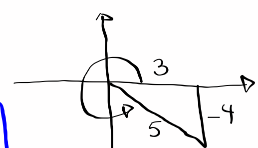
$\sin 2A = 2\sin A \cos A$

$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Given $\sin A = \frac{-4}{5}$, A is in Q IV Find

$$\sin 2A = 2\sin A \cos A = 2 \cdot \frac{-4}{5} \cdot \frac{3}{5} = \boxed{\frac{-24}{25}}$$



Q IV

$$\cos 2A = \cos^2 A - \sin^2 A = \left(\frac{3}{5}\right)^2 - \left(\frac{-4}{5}\right)^2 = \frac{9}{25} - \frac{16}{25} = \boxed{\frac{-7}{25}}$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A} = \frac{2\left(\frac{-4}{3}\right)}{1 - \left(\frac{-4}{3}\right)^2} = \frac{\frac{-8}{3}}{1 - \frac{16}{9}} = \boxed{\frac{24}{7}}$$

Jan 22-8:53 AM

Verify $\cot x \sin 2x - \cos 2x = 1$ ✓

$\cos 2x = \cos^2 x - \sin^2 x$

$= \boxed{2\cos^2 x - 1}$

$= 1 - 2\sin^2 x$

$\cot x \sin 2x - \cos 2x =$

$\frac{\cos x}{\cancel{\sin x}} \cdot 2\cancel{\sin x}\cos x - \cos 2x =$

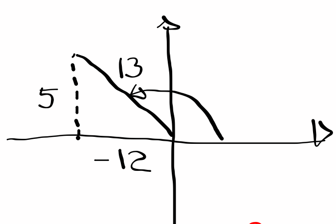
$2\cos^2 x - (2\cos^2 x - 1) =$

$\cancel{2\cos^2 x} - \cancel{2\cos^2 x} + 1 = 1$ ✓

Jan 22-9:02 AM

$\cos\theta = \frac{-12}{13}$, $\sin\theta > 0$, find $\tan 2\theta$

$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$



$= \frac{2\cdot\left(\frac{-5}{12}\right)}{1 - \left(\frac{-5}{12}\right)^2} = \boxed{\frac{-120}{119}}$

$\frac{\frac{-10}{12}}{1 - \frac{25}{144}} = \frac{\overset{12}{\cancel{144}}\cdot\frac{-10}{\cancel{12}}}{144\cdot 1 - \cancel{144}\cdot\frac{25}{\cancel{144}}} = \frac{-120}{119}$

LCD=144

Jan 22-9:07 AM

verify

$$\sin 2A \cos 2A = \boxed{\sin 2A} - 4\sin^3 A\cos A \checkmark$$

$$\sin 2A \cos 2A = \sin 2A\,(1 - 2\sin^2 A)$$

$$= \sin 2A - 2\boxed{\sin 2A}\sin^2 A$$

$$= \sin 2A - 2\cdot 2\sin A\cos A\cdot\sin^2 A$$

$$= \boxed{\sin 2A - 4\sin^3 A\cos A} \checkmark$$

Jan 22-9:15 AM

verify $\dfrac{1+\cos 2x}{\sin 2x} = \cot x \checkmark$

$$\frac{1+\cos 2x}{\sin 2x} = \frac{\cancel{1} + 2\cos^2 x \cancel{-1}}{2\sin x\cos x} = \frac{\cancel{2}\cos x\,\cancel{\cos x}}{\cancel{2}\sin x\,\cancel{\cos x}}$$

$$= \boxed{\cot x} \checkmark$$

Jan 22-9:22 AM

Do not Use Calculator to evaluate

1) $\cos^2\frac{\pi}{12} - \sin^2\frac{\pi}{12} = \cos^2 15^\circ - \sin^2 15^\circ$

$\cos^2 A - \sin^2 A = \cos 2A$

$= \cos 2\cdot 15^\circ = \cos 30^\circ = \boxed{\frac{\sqrt{3}}{2}}$

2) $1 - 2\sin^2 22.5^\circ = \cos 2\cdot 22.5^\circ = \cos 45^\circ = \boxed{\frac{\sqrt{2}}{2}}$

$1 - 2\sin^2 A = \cos 2A$

3) $2\cos^2 67.5^\circ - 1 = \cos 2\cdot 67.5^\circ = \cos 135^\circ$

$2\cos^2 A - 1 = \cos 2A$ $\quad = -\cos 45^\circ = \boxed{-\frac{\sqrt{2}}{2}}$

Jan 22-9:27 AM

Find x

$\sec 3x = \csc 2x$

when Cofunctions are equal,

Angles are Complementary

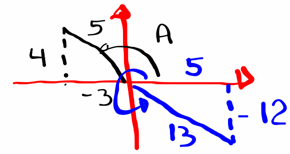
$3x + 2x = 90^\circ \qquad 5x = 90 \qquad \boxed{x = 18}$

Jan 22-9:37 AM

Class QZ

Given $\sin A = \frac{4}{5}$, $\frac{\pi}{2} < A < \pi$

$\cos B = \frac{5}{13}$, $\frac{3\pi}{2} < B < 2\pi$



Find $\sin(A+B)$. $\boxed{\text{Exact value}}$

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$
$$= \frac{4}{5}\cdot\frac{5}{13} + \frac{-3}{5}\cdot\frac{-12}{13}$$
$$= \frac{20}{65} + \frac{36}{65} = \boxed{\frac{56}{65}}$$

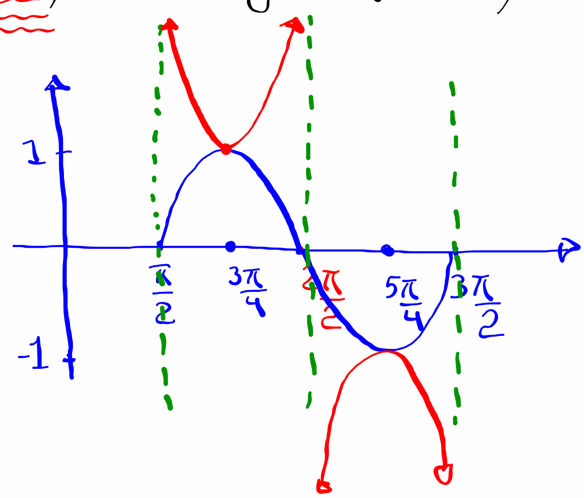
Jan 22-9:40 AM

Graph $y = \sin(2x - \pi)$ and $y = \csc(2x - \pi)$

$0 \le 2x - \pi \le 2\pi$

$\pi \le 2x \le 3\pi$

$\frac{\pi}{2} \le x \le \frac{3\pi}{2}$

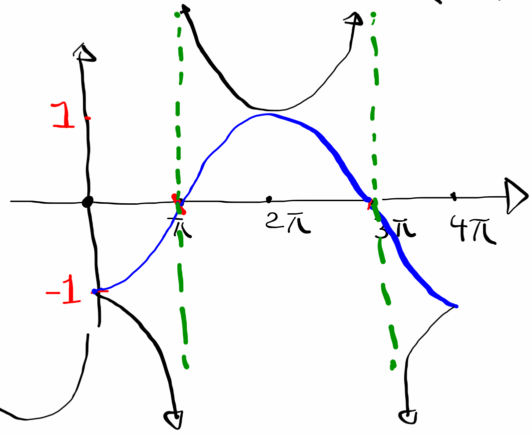


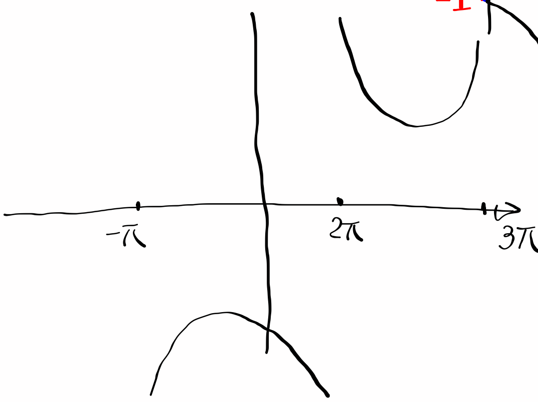
Jan 22-10:05 AM

Graph $y = -\cos\left(\frac{1}{2}x\right)$ and $y = -\sec\left(\frac{1}{2}x\right)$

$0 \le \frac{1}{2}x \le 2\pi$

$0 \le x \le 4\pi$



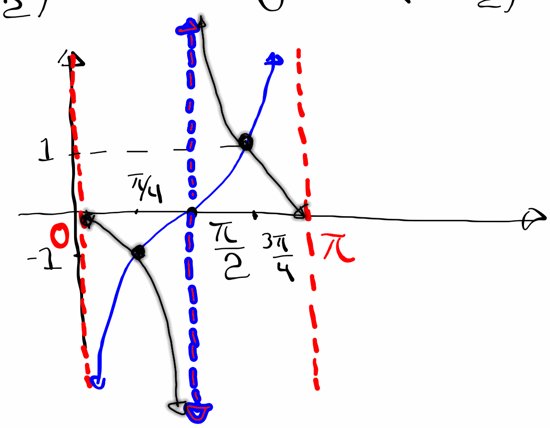


Jan 22-10:10 AM

Graph $y = \tan\left(x - \frac{\pi}{2}\right)$ and $y = \cot\left(x - \frac{\pi}{2}\right)$

$-\frac{\pi}{2} < x - \pi/2 < \frac{\pi}{2}$

$0 < x < \pi$



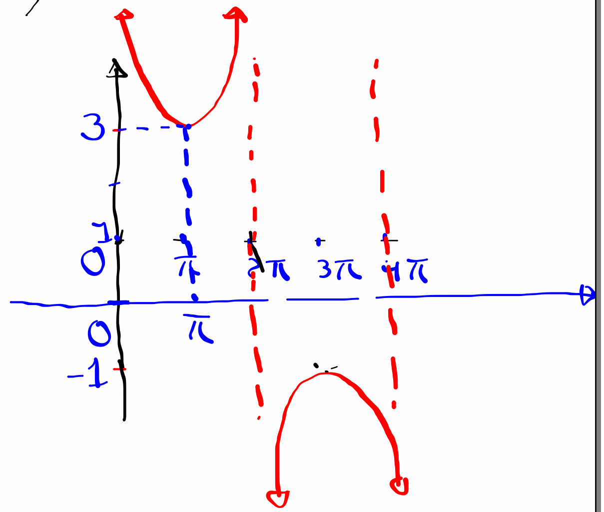
Jan 22-10:19 AM

Graph $y = 2\csc\left(\frac{1}{2}x\right) + 1$

1) Graph $y = 2\sin\left(\frac{1}{2}x\right)$

$0 \le \frac{1}{2}x \le 2\pi$

$0 \le x \le 4\pi$



Test Point

$x = \pi$

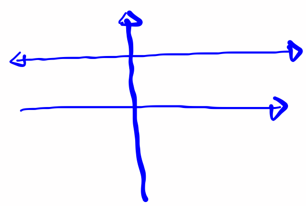
$y = 2\csc\left(\frac{1}{2}\pi\right) + 1$

$= 2 \cdot \csc\frac{\pi}{2} + 1 = 2 \cdot \frac{1}{\sin\frac{\pi}{2}} + 1 = 2 \cdot \frac{1}{1} + 1 = 3$

Jan 22-10:28 AM

Graph $y = \sin^2 x + \cos^2 x$

$y = 1$

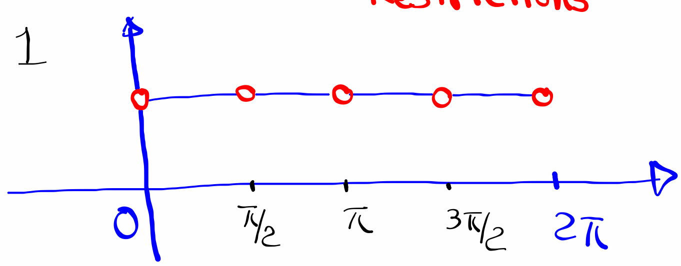


$y = 1$

Graph $y = \tan x \cot x$ for $0 \le x \le 2\pi$

Restrictions

$= 1$



$\tan 90^\circ$ und.

$\tan 270^\circ$ " .

$\cot x = \frac{\cos x}{\sin x}$

$\sin x = 0$ at

$0, \pi, 2\pi$

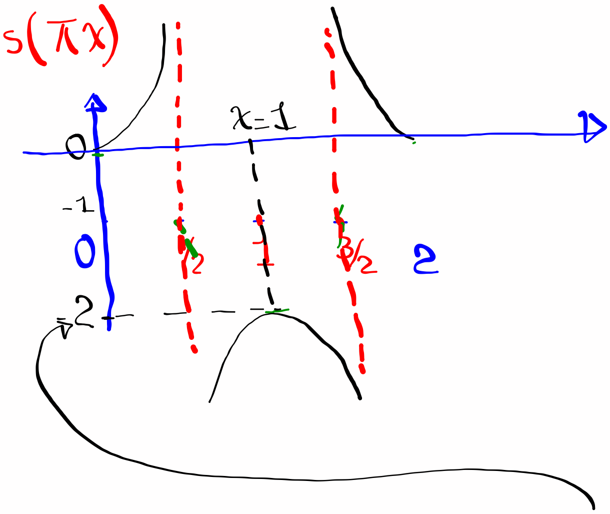
Jan 22-10:40 AM

Graph $y = \sec(\pi x) - 1$

Hint: Graph $y = \cos(\pi x)$

$0 \leq \pi x \leq 2\pi$

$0 \leq x \leq 2$



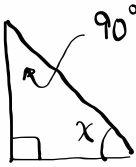
Test pt $x = 1$

$y = \sec(\pi \cdot 1) - 1$

$= \sec \pi - 1 = \frac{1}{\cos \pi} - 1 = \frac{1}{-1} - 1 = -1 - 1 = -2$

Jan 22-10:49 AM

Graph $y = \cot x$ using Cofunction & Compl. angle.



$90^\circ - x = \frac{\pi}{2} - x$

$y = \cot x = \tan\left(\frac{\pi}{2} - x\right) = \tan\left(-\left(x - \frac{\pi}{2}\right)\right)$

$-\frac{\pi}{2} < \frac{\pi}{2} - x < \frac{\pi}{2}$

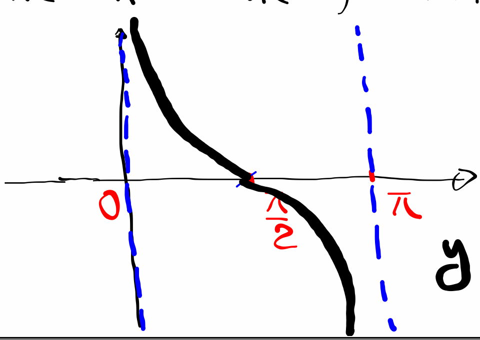
$= -\tan\left(x - \frac{\pi}{2}\right)$

Reflection

$-\frac{\pi}{2} < x - \frac{\pi}{2} < \frac{\pi}{2}$

$0 < x < \pi$

Recall $\tan(-\alpha) = -\tan \alpha$



$y = \cot x$

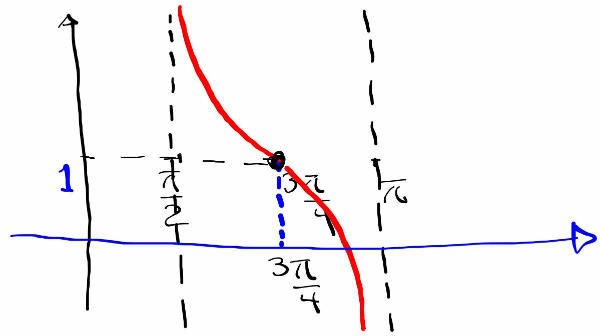
Jan 22-11:00 AM

Graph $y = \cot(2x - \pi) + 1$

$0 < 2x - \pi < \pi$

$\pi < 2x < 2\pi$

$\frac{\pi}{2} < x < \pi$



Test Point $x = \frac{3\pi}{4}$

$3\pi/2 - \pi =$

$y = \cot\left(\cancel{2}\cdot\frac{3\pi}{\cancel{4}\,2} - \pi\right) + 1$

$= \cot(\pi/2) + 1 = \frac{\cos \pi/2}{\sin \pi/2} + 1 = \frac{0}{1} + 1 = 0 + 1 = 1$

Jan 22-11:08 AM

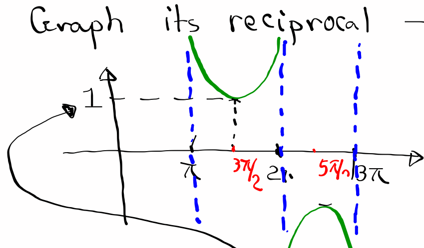
Graph $y = -\csc(-x + \pi)$

Some algebra $y = -\csc[-(x - \pi)]$

Recall $\csc(-\alpha) = -\csc\alpha$ $\quad y = - \cdot - \csc(x - \pi)$

Some prealgebra $y = \csc(x - \pi)$

Graph its reciprocal $\rightarrow y = \sin(x - \pi)$

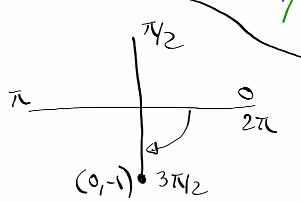


$0 \le x - \pi \le 2\pi$

$\pi \le x \le 3\pi$

$y = -\csc\left(\frac{-3\pi}{2} + \pi\right)$

$= -\csc\left(\frac{-\pi}{2}\right)$



$= -\frac{1}{\sin(-\frac{\pi}{2})} = -\frac{1}{-1} = 1$

Jan 22-11:19 AM

$y = -\sec(-.25\pi x) + 1 \Rightarrow y = -\sec(.25\pi x) + 1$

Reflection

$\boxed{\sec(-\alpha) = \sec\alpha}$

$\sin(-\alpha) = -\sin\alpha$

$\cos(-\alpha) = \cos\alpha$

$\tan(-\alpha) = -\tan\alpha$

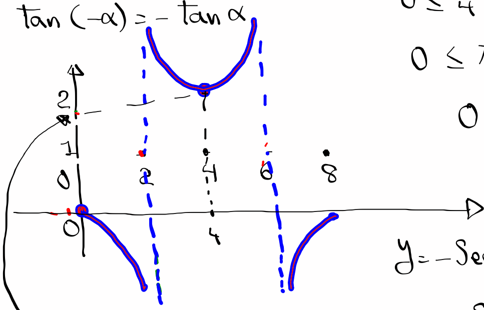
Graph $y = \cos(.25\pi x)$

$0 \le .25\pi x \le 2\pi$

$0 \le \frac{1}{4}\pi x \le 2\pi$

$0 \le \pi x \le 8\pi$

$0 \le x \le 8$



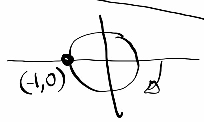
Test Point $x = 4$

$y = -\sec(-.25\pi \cdot 4) + 1$

$= -\sec(-\pi) + 1$

$= -\dfrac{1}{\cos(-\pi)} + 1$

$= -\dfrac{1}{-1} + 1 = 1 + 1 = 2$



Jan 22-11:31 AM

$\sin(A+B) = \sin A \cos B + \cos A \sin B$

$\sin(A-B) = \sin A \cos B - \cos A \sin B$

Add them

$\sin(A+B) + \sin(A-B) = 2\sin A \cos B$

$$\boxed{\sin A \cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]}$$

Jan 22-11:45 AM

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$

Add them

$$\cos(A+B) + \cos(A-B) = 2\cos A \cos B$$

$$\boxed{\cos A \cos B = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]}$$

Jan 22-11:49 AM

New identities:

Product - to - Sum

$\cos A \cos B$

$\sin A \sin B$

$\sin A \cos B$

Sum - to product

$\sin A + \sin B$

$\sin A - \sin B$

$\cos A + \cos B$

$\cos A - \cos B$

Jan 22-11:52 AM

Half-Angle Identities:

$\sin \frac{A}{2}$

$\cos \frac{A}{2}$

$\tan \frac{A}{2}$

Jan 22-11:55 AM

class QZ

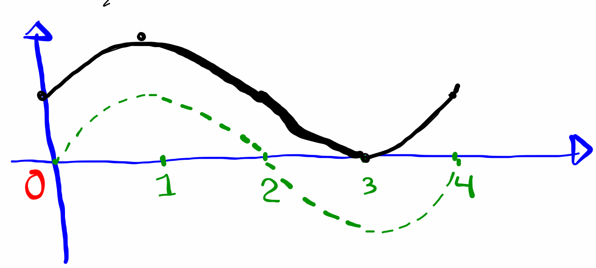
Graph $y = \sin(.5\pi x) + 1$

$0 \le .5\pi x \le 2\pi$

$0 \le \frac{1}{2}\pi x \le 2\pi$

$0 \le \pi x \le 4\pi$

$0 \le x \le 4$



Jan 22-11:56 AM